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Strength and ductility demand in relation to velocity spectrum

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ABSTRACT

Recent analytical studies on filtered earthquakes such as the Mexico City 1985 event have shown that inelastic displacements can be significantly greater than the elastic ones. It is found that the response of a yielded structure is highly dependent on its strength, the initial structural period, and the characteristics of the velocity spectrum of the ground motion. An equation relating inelastic response to the spectral velocity spectrum is made based on energy principles to explain both the equal- and unequal-displacement phenomena.

INTRODUCTION

Current practice in seismic-resistant design allows structures to yield during severe earthquakes provided that sufficient ductility is available for the large deformations. By assuming pure plastic behaviour in the inelastic range, the response of the structure can be idealized as a bilinear force-displacement plot as shown in Fig. 1.

It has been shown that, for a structure of any strength, the maximum inelastic displacement during an earthquake is very close to the displacement that would have been attained if the structure had remained elastic (Blume, Newmark, Corning, 1961). This idea is generally accepted and is incorporated in most design codes including the National Building Code of Canada 1990.

The "equal-displacement" phenomenon, however, is based on response to typical earthquakes that have very low predominant periods of vibration. Recent earthquakes in Mexico City and Loma Prieta have demonstrated the damaging effects of filtered earthquakes. This type of earthquake has its predominant period shifted to higher values by the local soil conditions. The acceleration response spectra for Taft S69E 1952, representing a typical earthquake, and for Mexico City SCT EW 1985 are shown together in Fig. 2. The effects of these different characteristics on the inelastic

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response of a structure will be examined, with the hope of developing some general concepts of inelastic response to earthquake excitations.

INELASTIC RESPONSE ENVELOPES

In the lateral load-displacement plot for any given structure, the line joining the maximum displacements at different yield levels is the inelastic response envelope (see Fig. 3). It indicates the inelastic behaviour of the structure having that particular initial period during that particular earthquake. Therefore, the envelope is a function of the initial structural period and of the characteristics of the earthquake. As mentioned earlier, for typical "normal" earthquakes, the envelope is defined by an almost vertical line. This is demonstrated by the envelopes in Fig. 4 obtained for the Taft ground motion using the dynamic program DRAIN-2D (Kanaan and Powell, 1973) on elasto-plastic single-degree-of-freedom systems. In this diagram, the results for each structural period are normalized with respect to the corresponding elastic values of load and displacement. Note that all the envelopes are very close to each other and stay reasonably vertical.

When the same dynamic analysis is carried out using the Mexico City ground motion, a different picture emerges. Figure 5 shows the resulting normalized inelastic response envelopes. It can be seen that not all the envelopes are vertical; in fact, they can be vertical, sloping outward, or sloping inward towards the elastic response line.

In Fig. 4, the line OA represents elastic behaviour. A line OB at a slope equal to that of OA divided by μ defines the maximum displacement reached by structures of ductility μ. In the idealized case, when the response envelope is a straight vertical line, it is evident that a structure of any initial period must be designed to yield at C in order to achieve a particular ductility of μ. It is also evident from the geometry that the "Force Reduction Factor", R, by which the elastic load at A must be reduced to give the design load at C is equal to μ.

The situation is clearly very different in Fig. 5, representing the Mexico City earthquake. The Force Reduction Factor is no longer equal to the ductility; indeed for the point A on that figure, one would have to use an R of about 2 to achieve a ductility of 4.

An explanation for the differences in the inelastic response envelopes can be found in the behaviour of an inelastic structure in relation to the characteristics of the earthquake. As the structure yields, it becomes less stiff, so that its fundamental period of vibration increases. Depending on the nature of the earthquake, then, it imparts more or less energy to this changed structure; if more, the ductility demand will be high; if less, the ductility demand will be lower. This phenomenon was observed in the Mexico City earthquake, and the increase in energy imparted to structures was related by Mitchell (1987) to the rising acceleration response spectrum, reproduced here as Fig. 6. It will be shown in the next section, however, that the velocity response spectrum is a more direct indicator of the ductility demand.

INELASTIC RESPONSE ENVELOPES FROM VELOCITY SPECTRA

The relationship between the Force Reduction Factor and the desired ductility depends upon the inelastic response envelope, as shown by Fig. 5. Therefore a method of determining the position of the latter would be useful.

As shown in Fig. 7, the response of an inelastic structure is represented as a bilinear curve. An "equivalent elastic" structure can be defined by a straight line of slope less than the actual stiffness, reaching the same maximum deflection as the inelastic structure. The slope of this line, or the "effective stiffness" of the equivalent elastic structure, is obtained by setting the area under the curve equal to that under the inelastic curve. The period of this equivalent elastic structure then suggests the period shift, as the real structure becomes inelastic in the earthquake.

The areas referred to above represent the maximum energy of deformation stored in the structure during the earthquake; by conservation of energy, this is essentially equal to the maximum kinetic energy achieved by the system, which is proportional to the square of the velocity. Thus, the load-displacement diagrams are indicative of the maximum velocities reached by the systems.

Applying these relationships to the original elastic structure and the equivalent elastic structure, and noting the relationship of the latter to the inelastic structure, one is able to express the Force Reduction Factor in terms of the ductility and the spectral velocities before and after the period shift:

$$R = \frac{V_1}{V_2} \sqrt{2\mu - 1} \qquad (1)$$

This equation shows the influence of the velocity response spectrum on the inelastic response envelope. If the velocity response spectrum is horizontal, so that V_1 equals V_2, the inelastic response envelope is given by curve H on Fig. 8, and the Force Reduction Factor is given by

$$R = \sqrt{2\mu - 1} \qquad (2)$$

This is the well-known equal-energy criterion; it implies that the energy under the inelastic force-displacement curve is equal to that under the original elastic structure curve. If the velocity response spectrum slopes downwards, as it does for "typical" earthquakes, then $V_2 < V_1$, the Force Reduction Factor is increased, and the inelastic response envelope is seen to lie closer to the initial stiffness line. The equal-displacement criterion applies to a line of this type. If the spectrum slopes upwards, as it frequently does when the ground motion has been filtered by local site conditions, then $V_2 > V_1$, R is reduced, and the inelastic response envelope moves further from the initial stiffness line, outside that for the equal energy case.

In order to apply these concepts, it is necessary to determine the period shift, so that V_1 and V_2 can be found. Turning again to Fig. 7, the stiffness of the equivalent elastic structure can be related to the initial stiffness and the ductility demand on the inelastic structure; this enables one to determine the period of this structure in terms of the initial period:

$$T_2 = \frac{\mu}{\sqrt{2\mu-1}} T_1 \qquad (3)$$

The period shift is given here in terms of the ductility demand; a similar relationship was previously presented by Iwan (1980), and a reasonable agreement between the two is shown on Fig. 9.

The validity of the equation for the determination of the inelastic response envelopes has been tested on three earthquake records: Taft S69E 1952, Mexico City SCT EW 1985, and an artificial strong ground shaking record for Richmond, British Columbia. Ductility values of 2 and 4 were used in the tests. The calculated Force Reduction Factors, Rc, were compared with the actual factors Ra, obtained by inelastic time-step analyses of elasto-plastic single-degree-of-freedom systems. The results were good when the velocity spectrum sloped downward, but when it sloped upwards the values were in error by a factor as large as 5.

An examination of Eq. 1 reveals that the effects of the change in structural damping during yielding, and of the actual shape of the velocity spectrum over the range of the period shift (instead of merely the two end values) have not been included. To account for these two effects, a modification factor is included:

$$R = f_N \frac{V_1}{V_2} \sqrt{2\mu-1} \qquad (4)$$

From a plot of the ratio of the calculated factors from Eq. 1 and the actual reduction factors against the secant slope of the velocity spectrum, the modification factor, f_N, is found to be a function of this slope and the ductility. Empirical values of f_N for ductilities of 2 and 4 were determined and are shown in Fig. 10. The limiting value of 1 for zero and negative slopes is imposed since no modification is required for horizontal and downward sloping spectra.

CONCLUSIONS

Studies of filtered earthquakes have revealed the possibility of non-vertical inelastic response envelopes occurring for structures with low to medium periods. The most important type of envelopes are those that slope outwards as they demand a lower force reduction factor to achieve a certain ductility in the structure. This force reduction factor can be estimated by a simple formula once the initial period, the ductility desired, and the characteristics of the velocity response spectrum are known.

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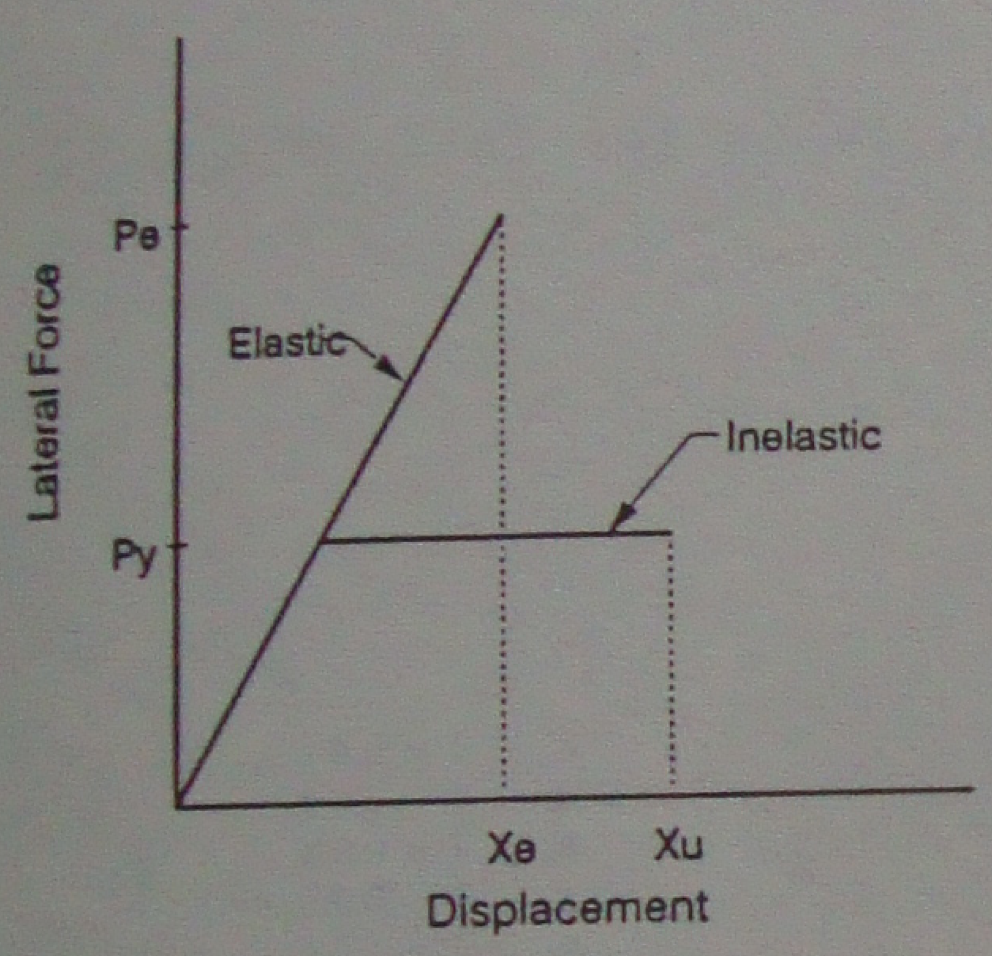

Figure 1. Seismic response of elasto-plastic structures.

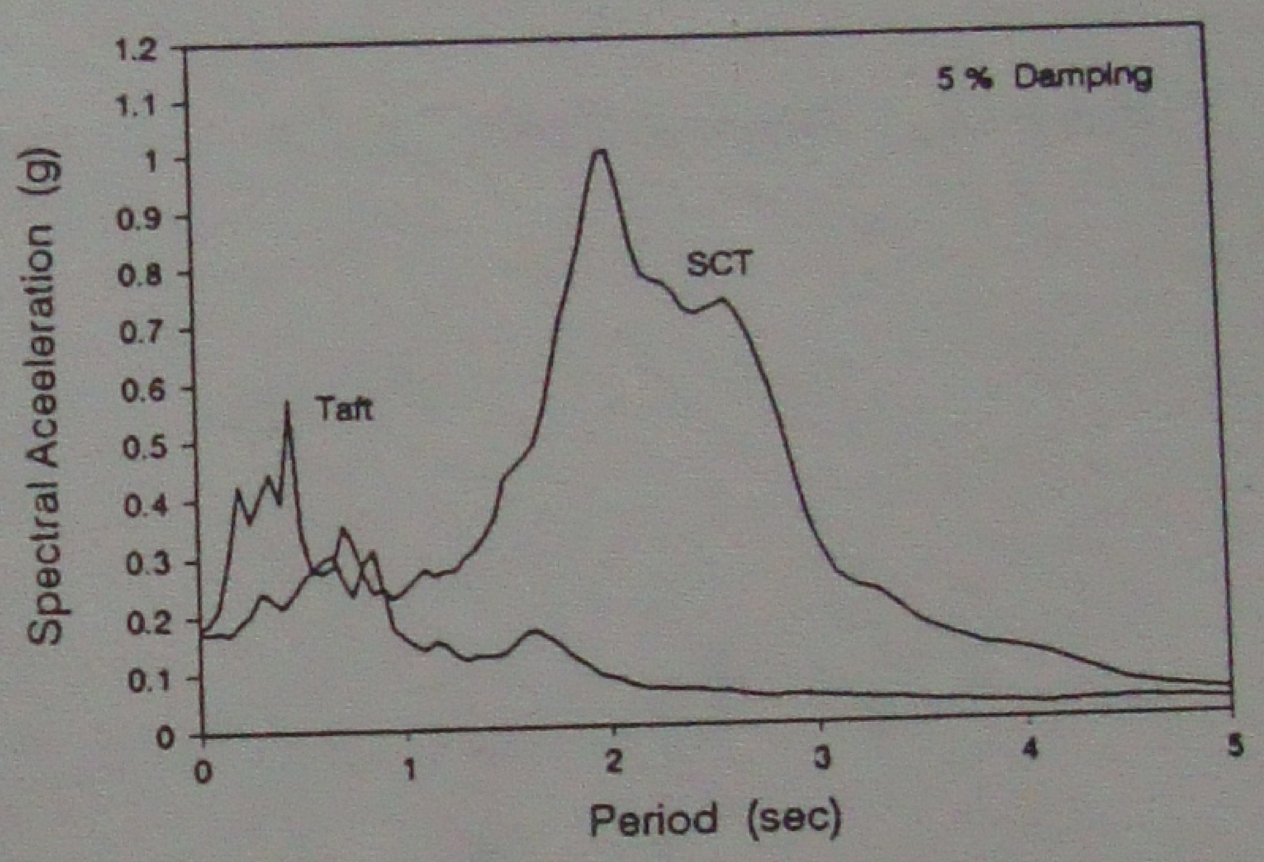

Figure 2. Sample acceleration response spectra.

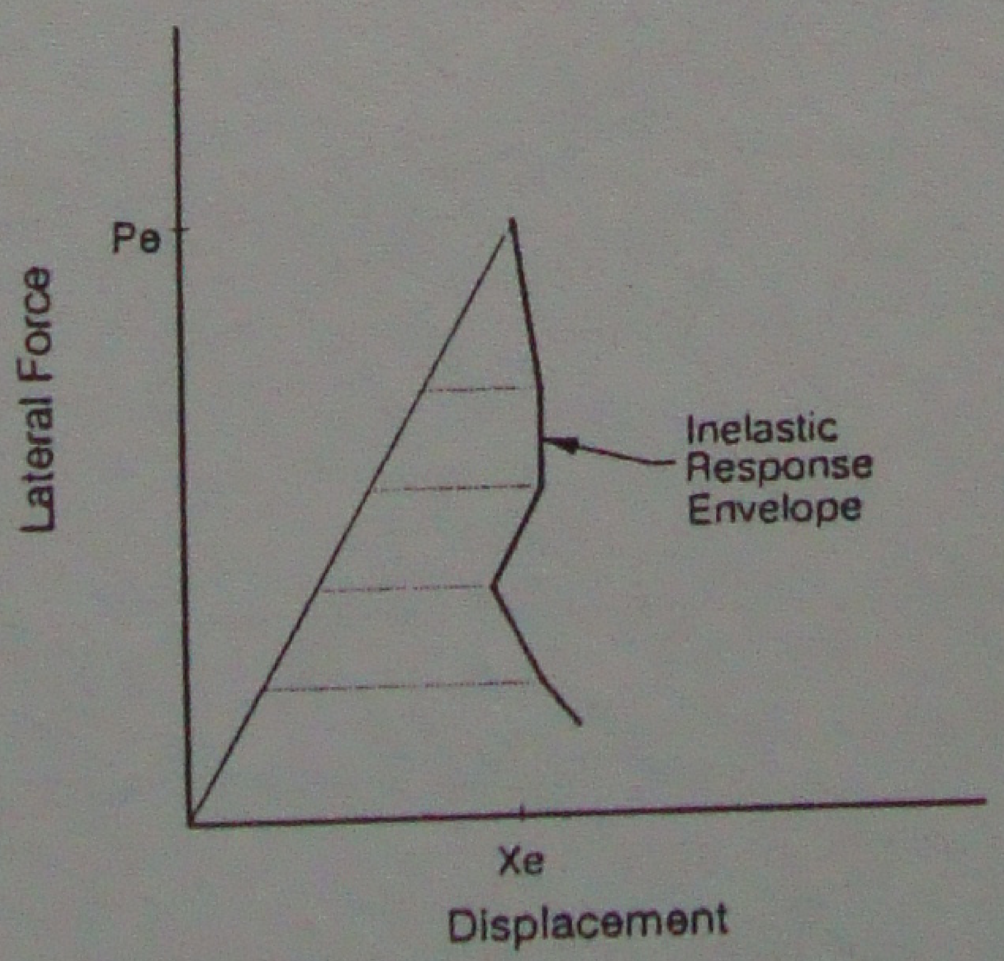

Figure 3. Inelastic response envelope.

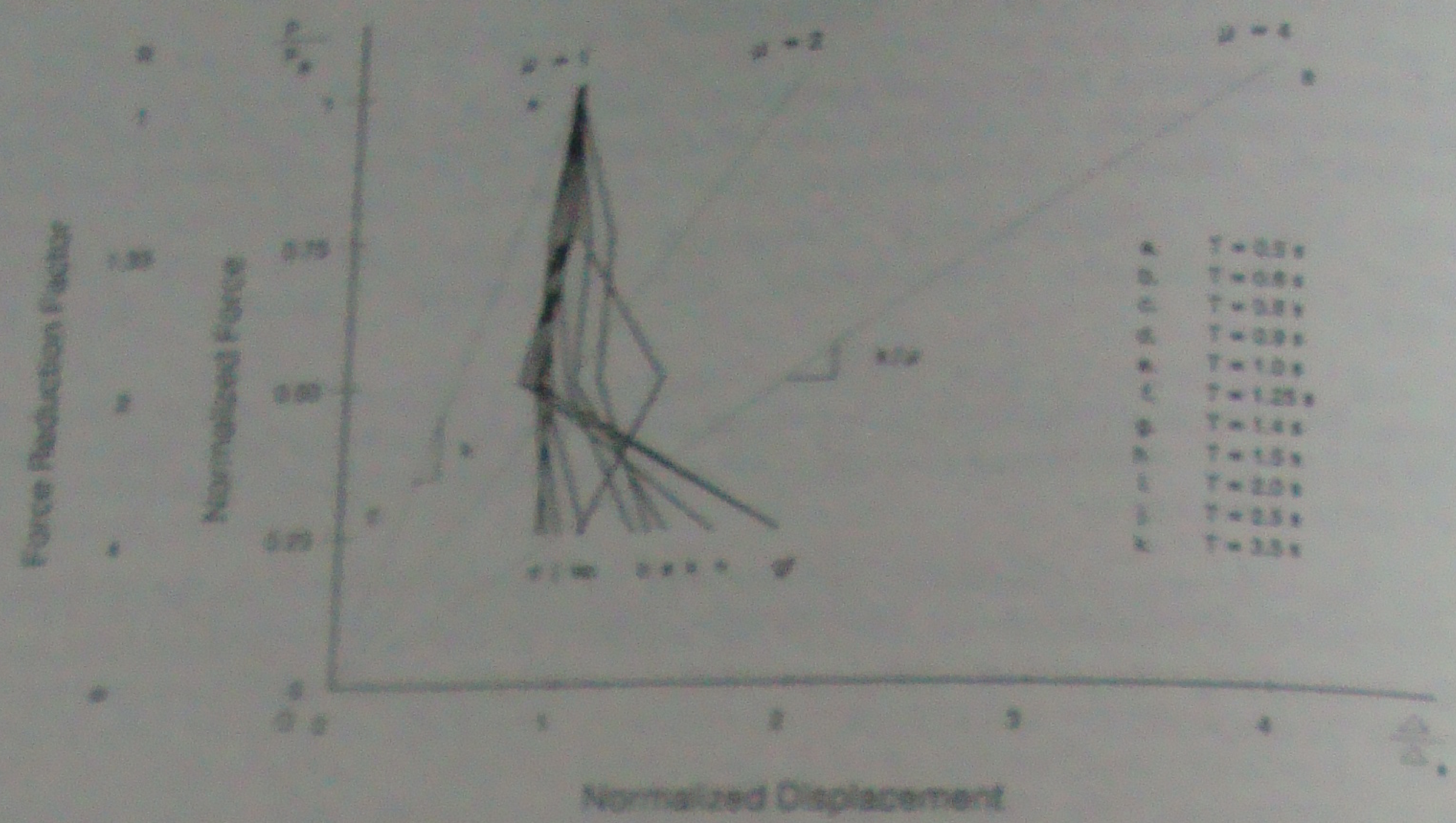

Figure 4. Normalized inelastic response envelopes for Taft S69E 1952.

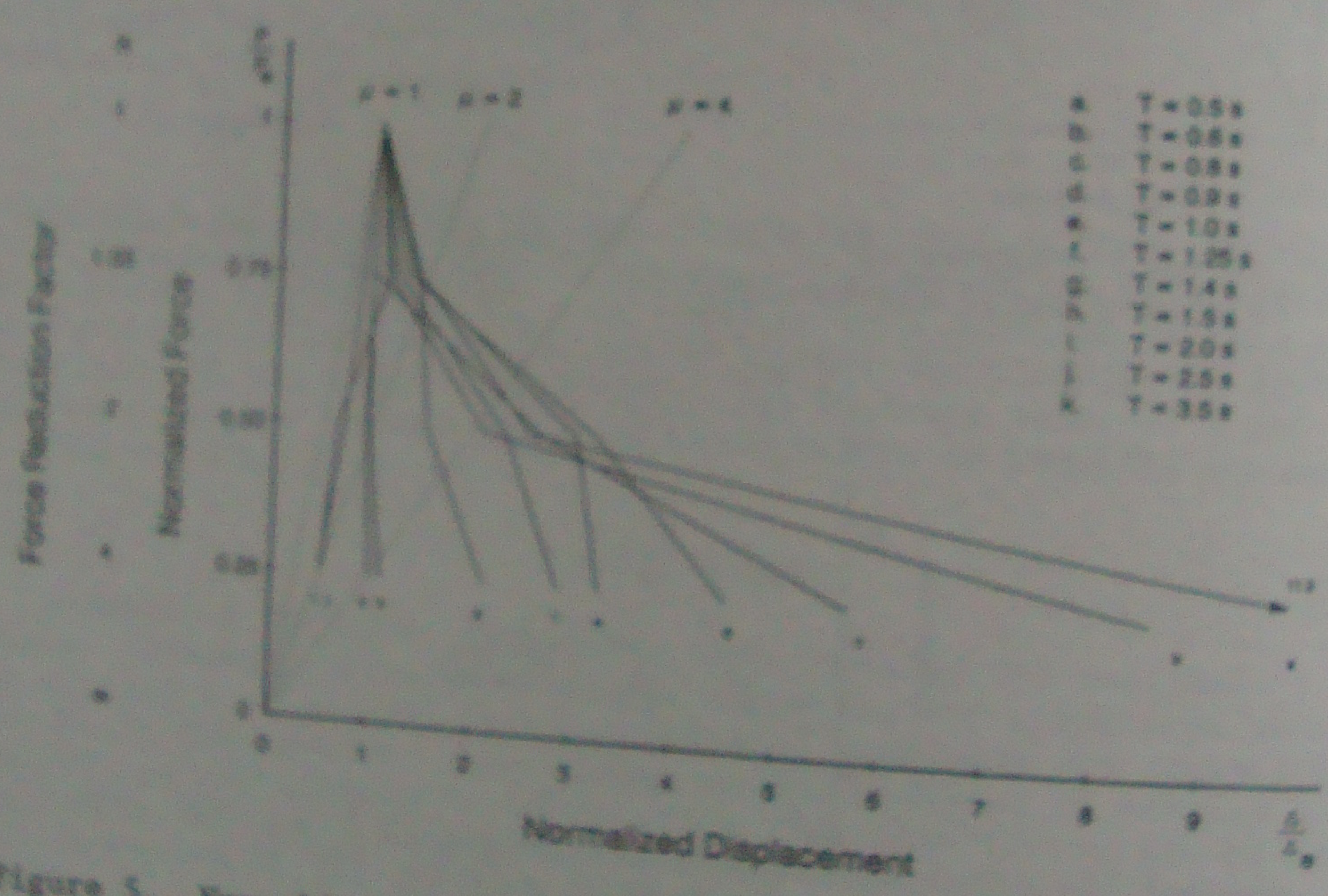

Figure 5. Normalized inelastic response envelopes for Mexico City SCT EW 1985.

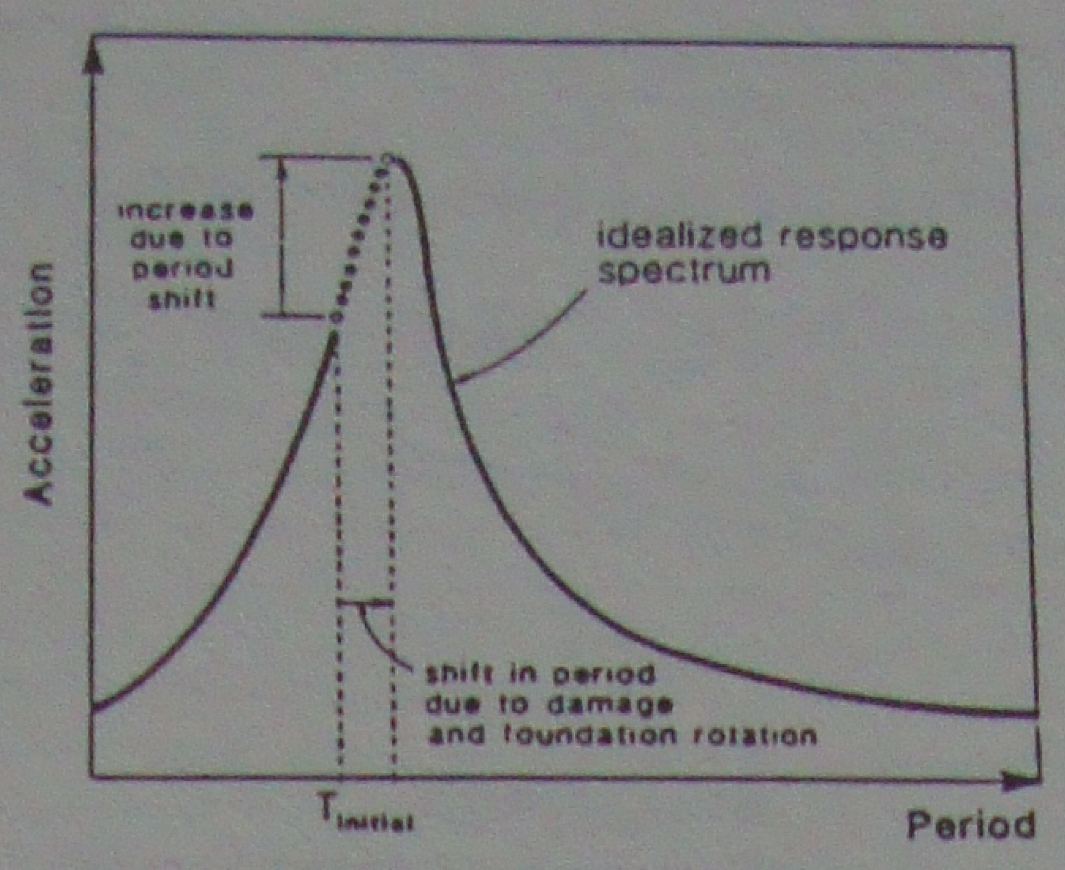

Figure 6. Energy increase from shift in period.

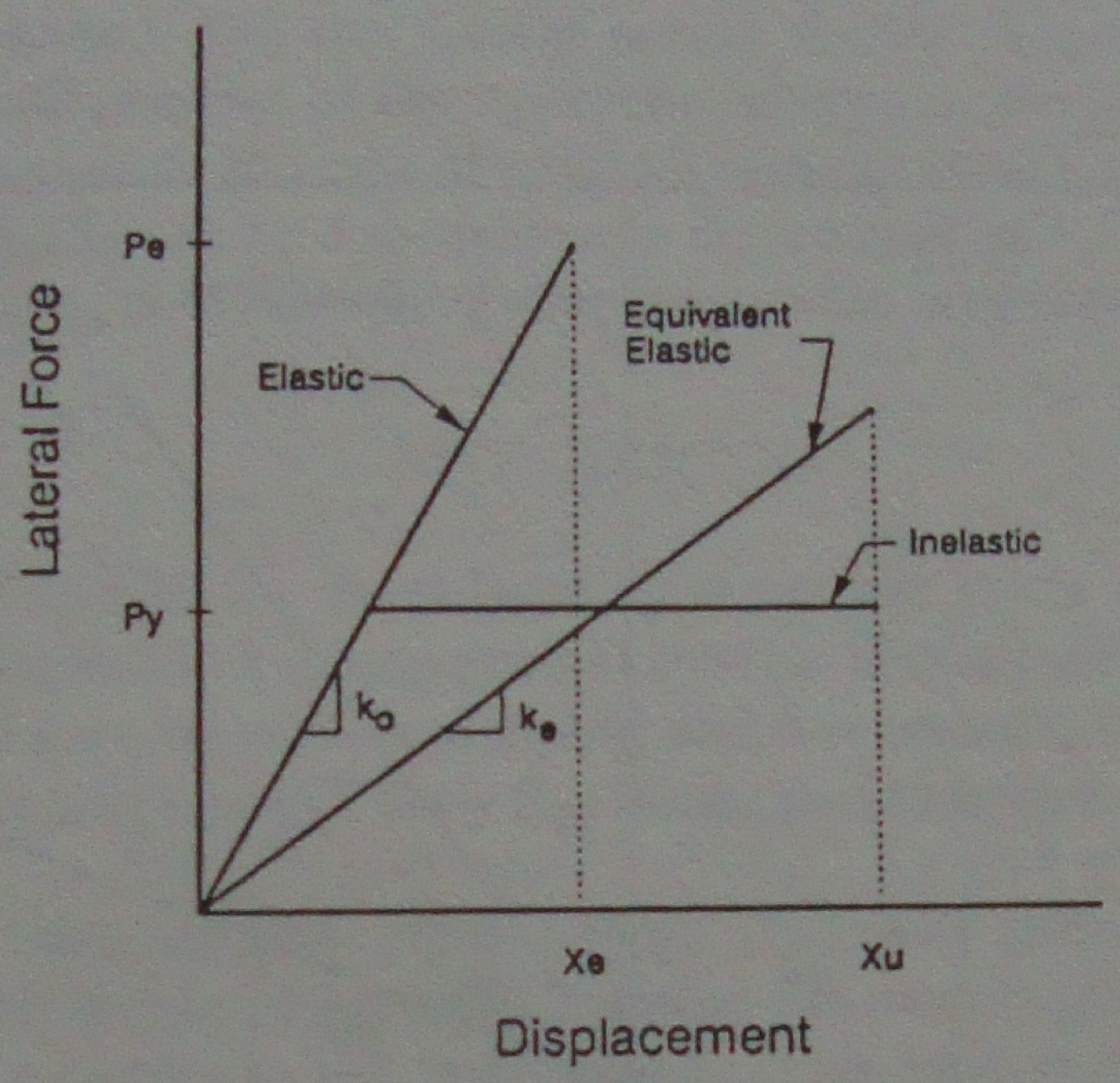

Figure 7. Equivalent elastic response.

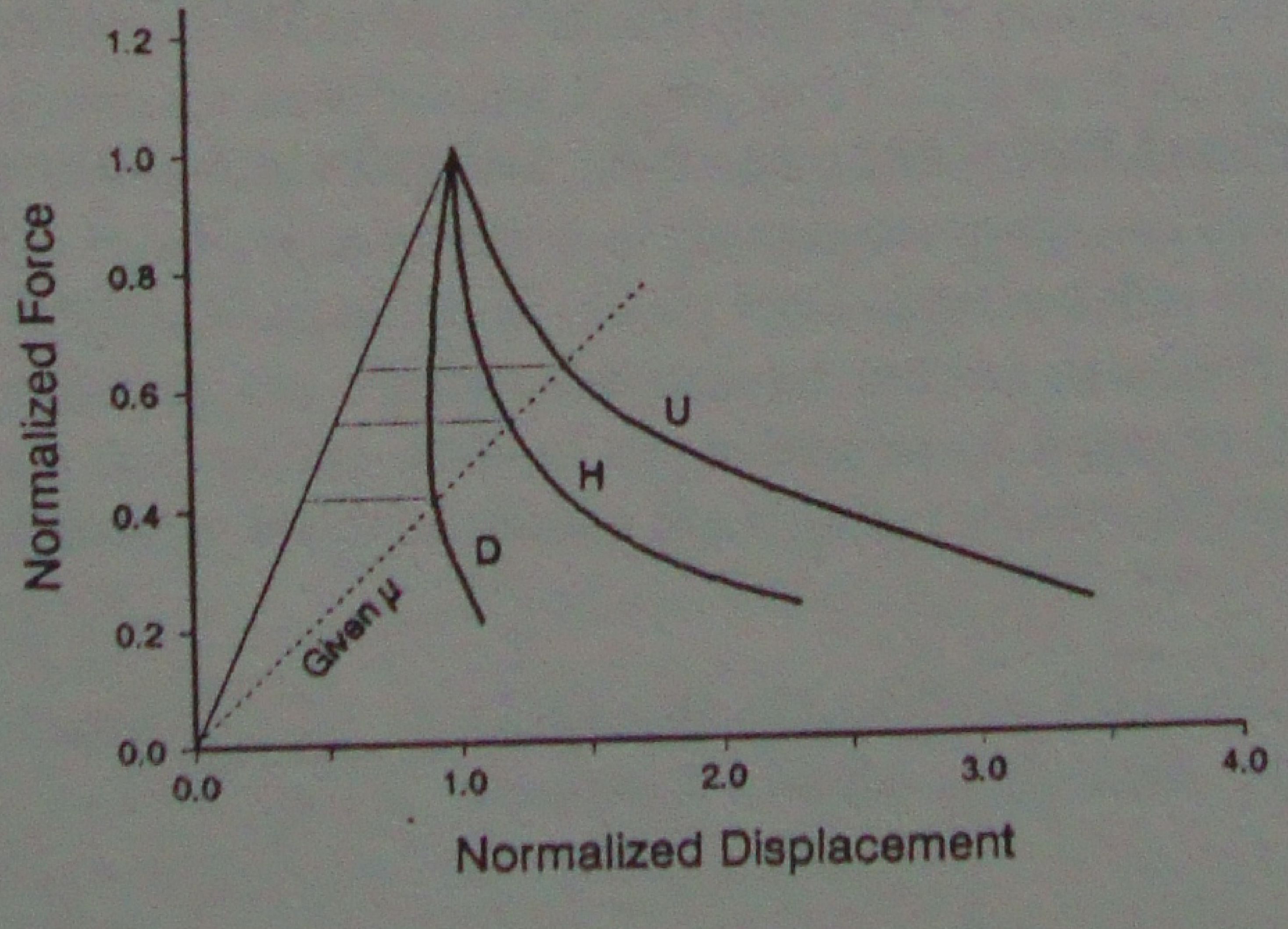

Figure 8. Types of inelastic response envelope.

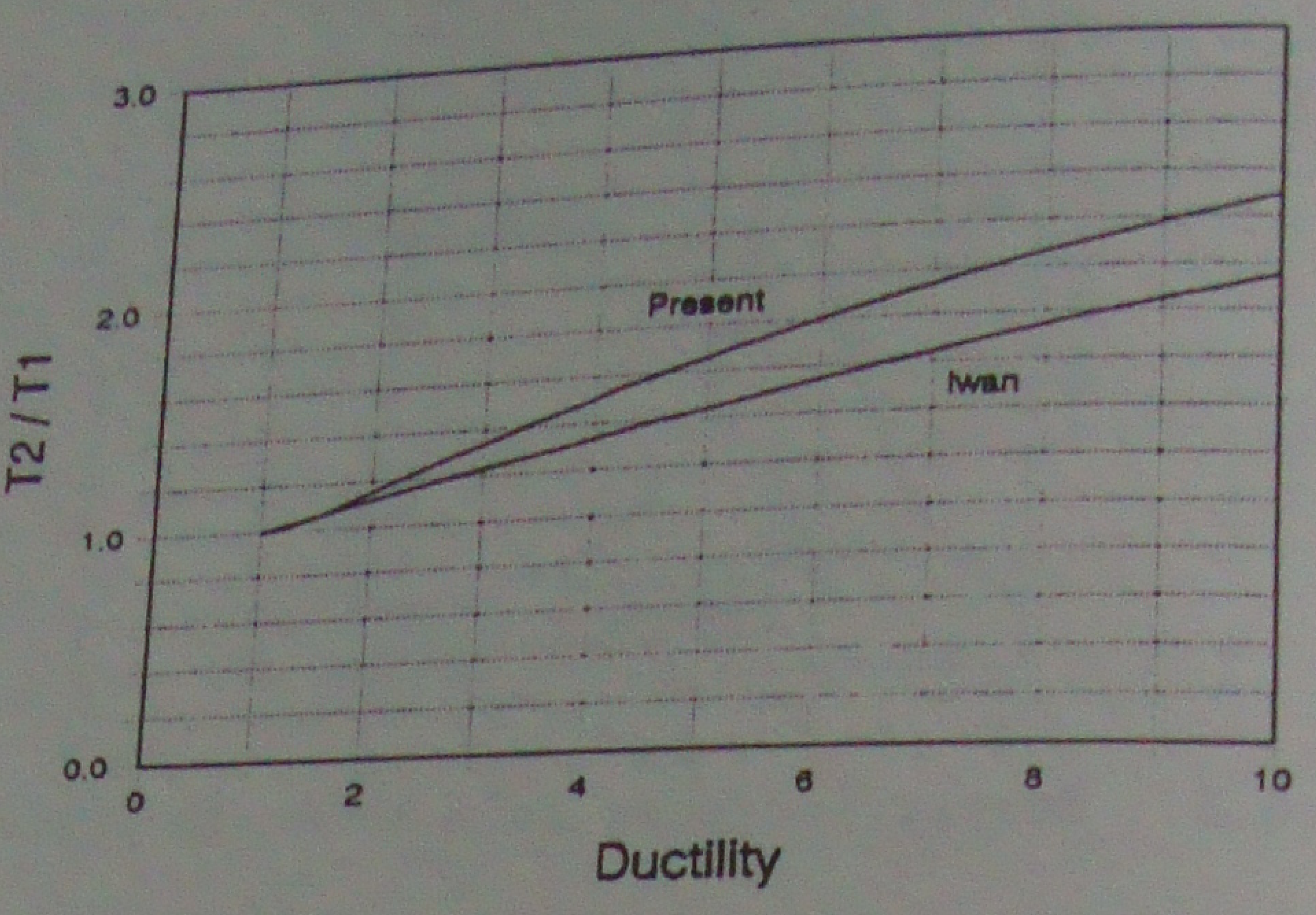

Figure 9. Ratio of equivalent to original periods of vibration.

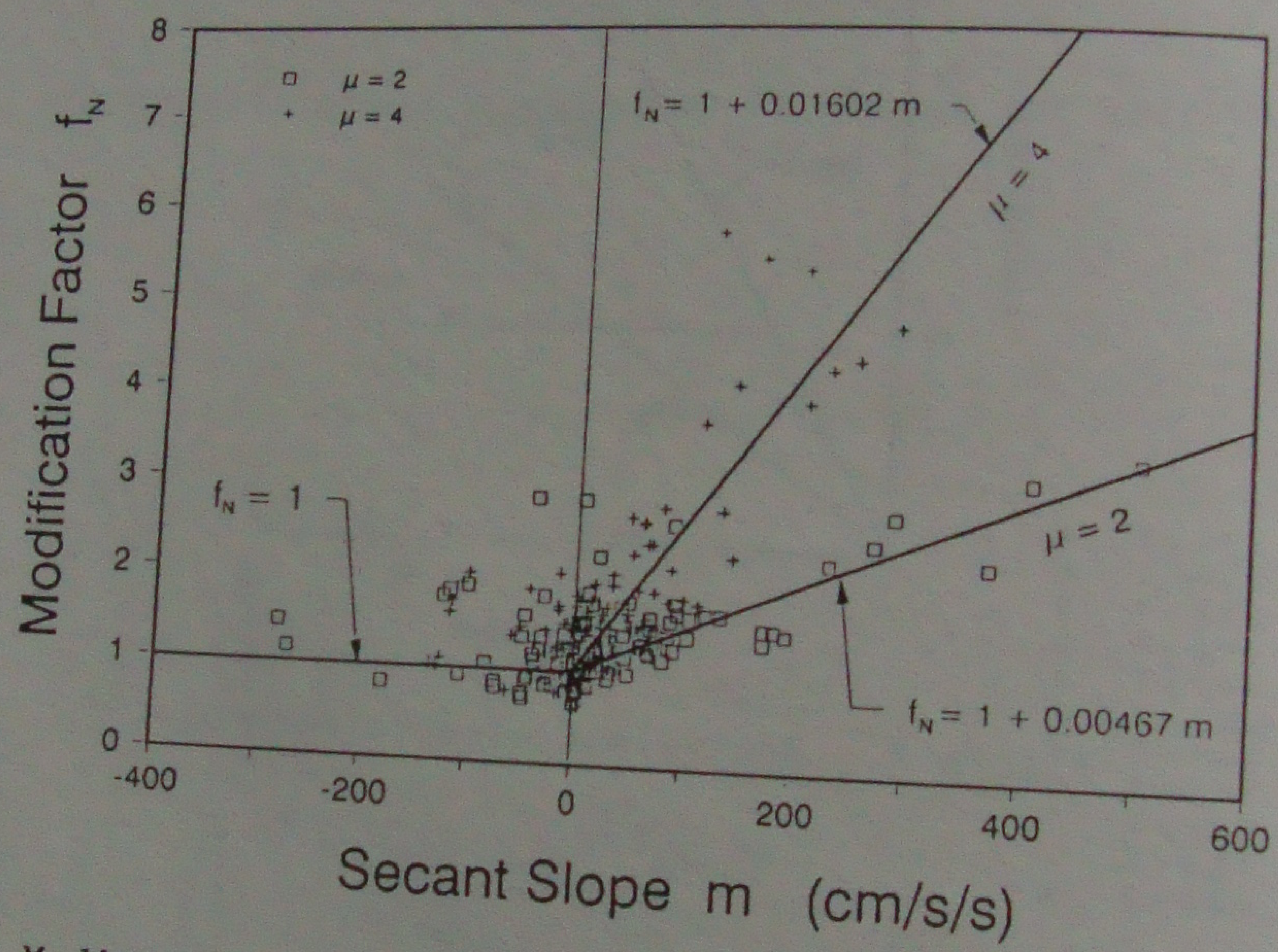

Figure 10. Modification factors for calculated force reduction factors.